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FRACTAL-TYPE EFFECTS IN COMPLEX SYSTEMS AT NUCLEAR SCALE

BY

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Abstract. Using the fractal theory of motion in the form of the Scale Relativity Theory in an arbitrary and constant fractal dimension, a fractal radioactive decay law and a fractal tunneling effect in complex systems at nuclear scale with spontaneous symmetry breaking properties are obtained.

Keywords: fractal theory of motion; complex systems; fractal radioactive decay law; fractal tunneling effect; spontaneous symmetry breaking.

1. Introduction

The notion of “complex system” is nowadays one of the inherent concept of modern science. It has long spread to a wider scale, having implications in sociological or cultural areas. A strict definition for this notion is hard to establishing, mainly due to the discovery or recognition of wider ranges of phenomena where it can be applied (Holovatch *et al*., 2017). Sometimes “complex system” refers to any system consisting of many interconnected parts which, as a whole, possesses properties that are not trivial aggregates of the properties of its separate parts (Sherington, 2010). This represents a fundamental characteristic of self-organisation and of the appearance of new

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properties (consciousness, just to give an example) often called emergence. A reference work for the collective effects of complex systems is the one written by Phillip Anderson (Anderson, 1972), a Nobel Prize Laureate in physics. The science of complex systems tries to establish the ways in which the constituent parts (or structural entities) give rise to the emerging collective behaviour of a whole system. But this interpretation is of limited use for physicists, because it encompasses too broad a set of circumstances.

However, some time ago a more useful definition has emerged: a system is complex if its behaviour essentially depends on its details (Parisi, 1999). Thus, this definition can be applied to phenomena such as deterministic chaos, quantum entanglement, protein folding etc. Structural disorder can influence the collective complex behaviour. Thus, an equilibrium state is difficult to reach and responses to external perturbations are slow and usually random (Parisi, 1999; Goldenfeld and Kadanoff, 1999). These different phenomenas are studied in different fields of physics such as dynamical systems, quantum mechanics and statistical physics. Their common property is that very small (infinitesimal) changes in initial conditions (even if different in nature) lead to radically different scenarios in their time evolution.

We must note now another important feature of complex systems: the macrostate and microstates dynamically update each other. This is due to the fact that, on the one hand, interactions between constituent parts lead to collective behaviour and define the macrostate but, on the other hand, the interactions are modified during the system's evolution and are influenced by the macrostate.

We must also take into consideration that the notion of complex systems implies that the interactions between constituents are time varying, and many different interaction types can be present at the same time. Interactions between structural entities can be very specific. These interactions change the states of the structural entities. Thus, we can say that the essence of many complex systems is that the states of their entities and interactions co-evolve over time (Holovatch *et al.*, 2017). The science of complex systems is therefore the generalisation of physics to forces and matter of a broader concept. Forces can be anything that change states of constituents, matter is anything where a force can be applied (Holovatch *et al*., 2017).

In this paper we propose some applications of the fractal theory of motion in the form of the Scale Relativity Theory in an arbitrary and constant fractal dimension for complex systems at nuclear scale.

2. Fractal-Type Radioactive Decay Law

Let us analyze Schrödinger's fractal equation in its stationary form (Nottale, 2011; Mercheș and Agop, 2016).

$$-2m_0\lambda^2\left(dt\right)^{\left(4/D_F\right)-2}\Delta\psi+U\psi=E\psi \tag{1}$$

where m_0 is the rest mass of the particle, λ is a coefficient associated to the fractal-non-fractal transition, dt is the scale resolution, D_F is the fractal dimension of the particle's motion curve, ψ is the fractal state function, U is the external scalar potential and E is the particle's energy.

In the case of a radial symmetry, $\psi=\psi\left(r\right)$, because Laplace's operator has the expression:

$$\Delta\psi=\frac{1}{r}\frac{d^2}{dr^2}\left(r\psi\right) \tag{2}$$

in the form

$$\varphi\left(r\right)=r\psi\left(r\right) \tag{3}$$

The Schrödinger's fractal Eq. (1) becomes:

$$-2m_0\lambda^2\left(dt\right)^{\left(4/D_F\right)-2}\frac{d^2\varphi\left(r\right)}{dr^2}+U\left(r\right)\varphi\left(r\right)=E\varphi\left(r\right) \tag{4}$$

In these conditions, the fractal probability current:

$$j\left(r\right)=-i\lambda\left(dt\right)^{\left(2/D_F\right)-1}\frac{1}{r^2}\left(\bar{\varphi}\frac{d\varphi}{dr}-\varphi\frac{d\bar{\varphi}}{dr}\right) \tag{5}$$

where $\bar{\varphi}$ is the complex conjugate of φ, induces through the surface of a sphere of radius r the total fractal probability current, $J\left(r\right)$,

$$J\left(r\right)=4\pi r^2 j\left(r\right)=4\pi i\lambda\left(dt\right)^{\left(2/D_F\right)-1}\left(\bar{\varphi}\frac{d\varphi}{dr}-\varphi\frac{d\bar{\varphi}}{dr}\right) \tag{6}$$

Let us now consider a spherical volume enclosed by a sphere of radius r_0. The fractal probability current integral $\rho=\bar{\psi}\psi$, where $\bar{\psi}$ is the complex conjugate of ψ, inside this volume is given by (Popescu, 2006):

$$P=\int_0^{r_0}\left(\bar{\psi}\psi\right)4\pi r^2 dr=4\pi\int_0^{r_0}\left(\bar{\varphi}\varphi\right)dr \tag{7}$$

Because the fractal probability conservation law imposes the relation

$$\partial_\tau P=-J\left(r_0\right)+J\left(0\right), \tag{8}$$

then through (7) we will have:

$$4\pi\frac{\partial}{\partial_{\tau}}\left[\int_{0}^{r_0}(\bar{\varphi}\varphi)dr\right]=4\pi i\lambda\left(dt\right)^{\left(2/D_F\right)-1}\left(\bar{\varphi}\frac{d\varphi}{dr}-\varphi\frac{d\bar{\varphi}}{dr}\right)\Big|_{0}^{r_0}=-J\left(r_0\right)+J\left(0\right) \quad (9)$$

The current $J(0)$ at $r=0$ can be null if there are no particles in the origin or sources. For example, in the absence of such a source, the following alternatives are possible (Gamov, 1928; Roy, 1986): i) temporal independence, situation in which the total fractal probability current $J(r_0)$ is null for any r_0; ii) temporal dependence, situation in which the total fractal probability current $J(r_0)\neq 0$ varies in with time so that $\bar{\varphi}\varphi$ can be integrated from $r=0$ to $r=r_0$. The second alternative can be used to describe dynamics involved in particle emission by nuclei (Fig. 1).

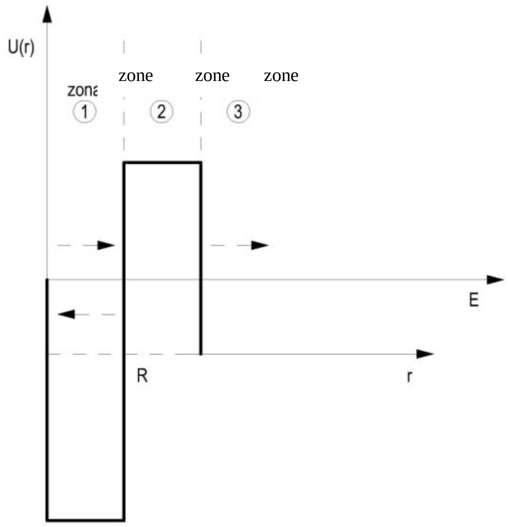

Fig. 1 – $U=U(r)$ dependency in particle emission by nuclei.

The fractal functions amplitudes for zones **1** and **3** are obtained by standard means using Schrödinger's fractal equation in its stationary form. We get

$$\varphi_1(r)=r\psi_1(r)=A_1\exp(ik_0r)+B_1\exp(-ik_0r) \quad (10)$$

for zone **1**, and

$$\varphi_3(r)=r\psi_3(r)=A_3\exp(ikr) \quad (11)$$

for zone **3** respectively.

Because the transmission coefficient, T, for all particle emitting nuclei is very low, then $A_1 \approx B_1$. In these circumstances, Eq. (7) becomes:

$$P = 4\pi\int_0^R (\bar{\varphi}\varphi)dr \approx 4\pi\left(2|A_1|^2 R\right) \tag{12}$$

where R defines here the nucleus radius.

Now the total fractal probability current outside the nuclear barrier becomes:

$$J_3 = -4\pi i\lambda\left(dt\right)^{\left(2/D_F\right)-1}|A_3|^2\left(2ik\right) = 4\pi v_3|A_3|^2 \tag{13}$$

with

$$v_3 = 2\lambda\left(dt\right)^{\left(2/D_F\right)-1}k \tag{14}$$

the velocity in zone **3**. Taking into account the conservation law (8) we will have:

$$2R\frac{d}{d\tau}|A_1|^2 = -v_3|A_3|^2 \tag{15}$$

from where, by multiplying both terms of the above relation with $1/|A_1|^2$, it results:

$$\frac{d|A_1|^2}{|A_1|^2} = -\frac{v_3}{2R}\frac{|A_3|^2}{|A_1|^2}d\tau \tag{16}$$

Since the fractal probability of a particle existing inside a nucleus is proportional with $|A_1|^2$, Eq. (16) leads us to the fractal probability variation with time P written as

$$\frac{dP}{P} = -\frac{v_3}{2R}\frac{|A_3|^2}{|A_1|^2}d\tau = -\gamma d\tau \tag{17}$$

i.e.

$$P(t) = P(0)\exp(-\gamma\tau) \tag{18}$$

with

$$\gamma = \frac{v_3}{2R}\frac{|A_3|^2}{|A_1|^2} = \frac{2\lambda\left(dt\right)^{\left(2/D_F\right)-1}k}{2R}\frac{|A_3|^2}{|A_1|^2} \tag{19}$$

the fractal disintegration constant. Therefore, relation (18) expresses the fractal-type radioactive decay law.

Relation (19) can be written in another way by introducing the barrier penetration coefficient *T*. For a rectangular barrier with different potential levels on the two sides, the penetration coefficient is given by:

$$T = \frac{v_3}{v_1}\frac{|A_3|^2}{|A_1|^2} \tag{20}$$

By substituting (20) in (19) we get:

$$\gamma = \frac{v_1}{2R}T \tag{21}$$

Since both v_1 and *T* are functionally dependent of the fractalization degree $G = \lambda(dt)^{\left(2/D_F\right)-1}$, *i.e.* $v_1 = v_1(G)$ and $T_1 = T_1(G)$, it results that the decay constant is dependent on the scale resolution.

3. Fractal Tunneling Effect in Physical Systems with Spontaneous Symmetry Breaking

In the fractal theory of physical systems with spontaneous symmetry breaking, because of the shape of the potential, Schrödinger's stationary fractal equation cannot be integrated precisely. Determining the fractal state functions and eigenvalues can only be made through successive approximations, in the framework of a possible fractal theory of stationary perturbations. In order to circumvent the mathematical difficulties generated by employing such a method, we will admit a simplifying hypothesis of the effective potential, as shown in Fig. 2.

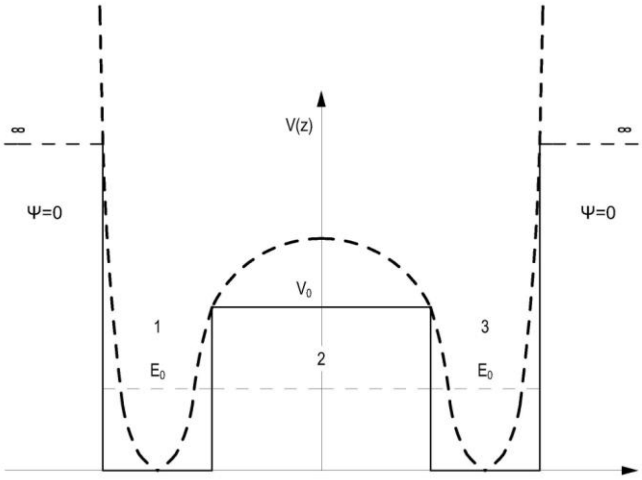

Fig. 2 – The effective potential for the case of a fractal tunneling effect for physical systems with spontaneous symmetry breaking.

So, let us admit that the potential for the spontaneous symmetry breaking case can be approximated to an effective potential, as shown in Fig. 2. In these conditions, Schrödinger's stationary fractal equation becomes (Dariescu *et al*., 2007):

$$\frac{a^2\psi_\alpha}{dz^2}+\frac{1}{2m_0\lambda^2\left(dt\right)^{\left(4/D_F\right)-2}}\left[E-V_\alpha\right]\psi_\alpha=0,\ \alpha=\overline{1,3} \tag{22}$$

For each of the three regions the solutions of the equations are (Dariescu *et al*., 2007):

$$\begin{aligned}\psi_1\left(z\right)&=C_+e^{ikz}+C_-e^{-ikz}\\ \psi_2\left(z\right)&=Be^{qz}+Ce^{-qz}\\ \psi_3\left(z\right)&=D_+e^{ikz}+D_-e^{-ikz}\end{aligned} \tag{23}$$

with

$$\begin{aligned}k&=\left[\frac{E}{2m_0\lambda^2\left(dt\right)^{\left(4/D_F\right)-2}}\right]^{1/2}\\ q&=\left[\frac{V_0-E}{2m_0\lambda^2\left(dt\right)^{\left(4/D_F\right)-2}}\right]^{1/2}\end{aligned} \tag{24}$$

and C_+, C_-, B, C, D_+, D_- integration constants.

Due to the infinite potential in the two extreme regions, $\left|z\right|>l$, the fractal state function continuity in $z=\pm l$ implies:

$$\begin{aligned}\psi_2\left(-l\right)&=C_+e^{-ikl}+C_-e^{ikl}=0\\ \psi_3\left(l\right)&=D_+e^{ikl}+D_-e^{-ikl}=0\end{aligned} \tag{25}$$

Since the states density $\left|\psi\right|^2$ is not altered by the multiplication of the fractal state function by a constant phase factor, the two equations for $C_\pm$ and $D_\pm$ can be immediately solved by imposing the mensuration:

$$\begin{aligned}C_+&=\frac{A}{2i}e^{ikl},\ C_-=-\frac{A}{2i}e^{-ikl}\\ D_+&=\frac{D}{2i}e^{-ikl},\ D_-=-\frac{D}{2i}e^{ikl}\end{aligned} \tag{26}$$

so that $\psi_{1,3}$ are given through simple expressions:

$$\begin{aligned}\psi_1(z) &= A\sin\left[k(z+l)\right]\\ \psi_3(z) &= D\sin\left[k(z-l)\right]\end{aligned} \tag{27}$$

These, along with ψ_2, lead to the concrete form of "alignment conditions" in $z=\pm d$

$$\begin{aligned}&\psi_1(-d)=\psi_2(-d),\ \psi_2(d)=\psi_3(d)\\ &\frac{d\psi_1}{dz}(-d)=\frac{d\psi_2}{dz}(-d),\ \frac{d\psi_2}{dz}(d)=\frac{d\psi_3}{dz}(d)\end{aligned} \tag{28}$$

namely

$$\begin{aligned}&e^{-qd}B+e^{qd}C = A\sin\left[k(l-d)\right]\\ &qe^{-qd}B-qe^{qd}C = kA\cos\left[k(l-d)\right] \text{ în } z=-d\\ &e^{qd}B+e^{-qd}C = -D\sin\left[k(l-d)\right]\\ &qe^{qd}B-qe^{-qd}C = kD\cos\left[k(l-d)\right] \text{ în } z=d\end{aligned} \tag{29}$$

Due to the algebraic form of the two equation pairs, in order to establish the concrete expression of the "secular equation" (for eigenvalues *E* of the energy), $\Delta[E]=0$, we avoid calculating the 4th order determinant, $\Delta\left[k(E),q(E)\right]$, formed with the fractal amplitude coefficients *A*, *B*, *C*, *D*, by employing the following: we note with ρ the relation *C*/*B* and we divide the first equation to the second one, for each pair. It results:

$$\begin{aligned}\frac{e^{2qd}\rho+1}{e^{2qd}\rho-1} &= -\frac{q}{k}tg\left[k(l-d)\right]\\ \frac{e^{-2qd}\rho+1}{e^{-2qd}\rho-1} &= \frac{q}{k}tg\left[k(l-d)\right]\end{aligned} \tag{30}$$

which leads to the equation for ρ:

$$\frac{e^{2qd}\rho+1}{e^{2qd}\rho-1}+\frac{e^{-2qd}\rho+1}{e^{-2qd}\rho-1}=0 \tag{31}$$

We find $\rho^2=1$ which implies

$$\rho_-=-1,\ \rho_+=1 \tag{32}$$

For $\rho_+ = 1$, the amplitude function, $\psi_2(z) \sim \coth(qz)$, is symmetric just as the fractal states of the system with regard to the (spatial) reflectivity against the origin. Then the permitted values equation of the energy of these states, E_S, has the concrete form:

$$\tan\left[k_S(l-d)\right] = -\frac{\coth(q_S d)}{q_S} k_S \tag{33}$$

where

$$k_S = \left[\frac{E_S}{2m_0\lambda^2 (dt)^{\left(4/D_F\right)-2}}\right]^{1/2}$$
$$q_S = \left[\frac{V_0 - E_S}{2m_0\lambda^2 (dt)^{\left(4/D_F\right)-2}}\right]^{1/2} \tag{34}$$

For $\rho_- = -1$, the amplitude function, $\psi_2(z) \sim \tanh(qz)$, so that the states will be antisymmetric and permitted values equation, E_A, becomes:

$$\tan\left[k_A(l-d)\right] = -\frac{\tanh(q_A d)}{q_A} k_A \tag{35}$$

where

$$k_A = \left[\frac{E_A}{2m_0\lambda^2 (dt)^{\left(4/D_F\right)-2}}\right]^{1/2}$$
$$q_A = \left[\frac{V_0 - E_A}{2m_0\lambda^2 (dt)^{\left(4/D_F\right)-2}}\right]^{1/2} \tag{36}$$

It results, for now, at least qualitatively, that the presence of the barrier (of finite height V_0) between *-d* and *d*, leads to the splitting of the fundamental level E_0 into two sublevels E_S, E_A accounting for the two types of states – symmetric and antisymmetric, respectively, in which the system can be found. Because both eigenvalues equations are strongly transcendent, a direct estimation of solutions $E_{S,A}$ could be possible only by means of numerical methods. More precisely, we can see here a process of coupling between two different fractal states, made possible through a fractal tunneling effect.

4. Conclusions

In the fractal theory of motion in the form of the Scale Relativity Theory in an arbitrary and constant fractal dimension for complex systems at nuclear scale, some particular dynamics are analyzed. In this context, we obtain the expressions for a fractal radioactive decay law and also for a fractal tunneling effect, for complex systems with spontaneous symmetry breaking properties. These results can be applied to various technological fields, such as nanorobotics and nanomaterials engineering (Agape *et al*., 2016; Agape *et al*., 2017; Gaiginschi and Agape, 2016; Gaiginschi *et al*., 2011; Gaiginschi *et al*., 2014a; Gaiginschi *et al*., 2014b; Gaiginschi *et al*., 2017; Vornicu *et al*., 2017).

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EFECTE DE TIP FRACTAL ÎN SISTEME COMPLEXE LA SCARĂ NUCLEARĂ

(Rezumat)

Utilizând teoria fractală a mișcării sub forma Teoriei Relativităţii de Scală în dimensiune constantă și arbitrară se obține legea de dezintegrare radioactivă fractală și efectul tunel fractal în sisteme complexe cu rupere spontană de simetrie.